Hunters Hill High School

Student Number

2023 TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 100

Section I – 10 marks (pages 3-7)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8-25)

- Attempt Questions 11–31
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

---

1 Which inequality gives the domain of $y = \sqrt{4x - 5}$?

A. $x < \frac{5}{4}$

B. $x > \frac{5}{4}$

C. $x \geq \frac{5}{4}$

D. $x \leq \frac{5}{4}$

2 The function $f(x) = x^2$ is transformed into $g(x) = 3(x + 5)^2$ by a translation, followed by a dilation.

Which row of the table shows the direction(?) of the transformations?

	Translation	*Dilation*
A.	Horizontal	Vertical
B.	Vertical	Horizontal
C.	Vertical	Vertical
D.	Horizontal	Horizontal

3 After a fundraising event, Steven's school was in the top 5% of funds raised.

Assuming the funds raised by each school were normally distributed, what is the best approximation of the z-score for Steven's school?

A. 1.0

B. 2.0

C. 1.8

D. 1.3

4 Given $f(x) = 3x - \sin(3x)$, find $f'(x)$.

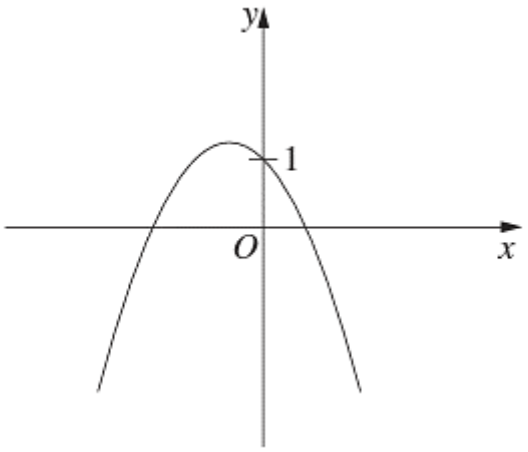
A. $f'(x) = 3 - \cos(3x)$

B. $f'(x) = 3 - 3\cos(3x)$

C. $f'(x) = 3 - 3\cos(x)$

D. $f'(x) = 3 + 3\cos(x)$

5 The graph of $y = ax^2 + bx + 1$ is drawn below:



Which row of the table shows the correct signs for a and b?

	$Sign\ of\ a$	$Sign\ of\ b$
A.	Positive	Negative
B.	Positive	Positive
C.	Negative	Negative
D.	Negative	Positive

6 What is $\int ex - e^x \, dx$?

A. $\frac{ex^2 - e^{2x}}{2} + c$

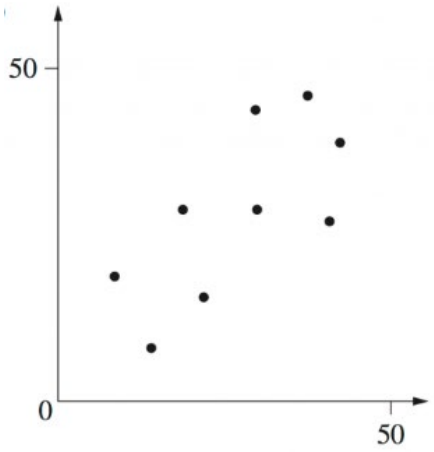
B. $\frac{ex^2}{2} - e^x + c$

C. $e - e^x + c$

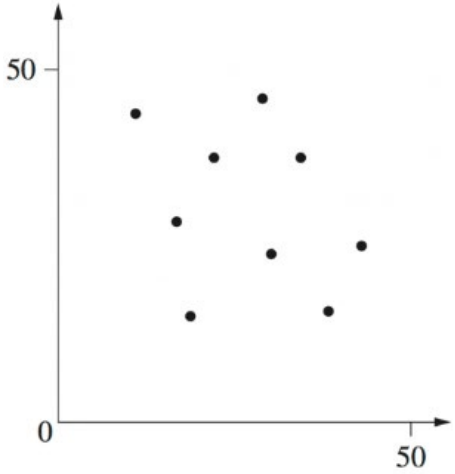
D. $\frac{ex^2}{2} + \frac{e^{x+1}}{x+1} + c$

7 Which graph best shows data with a correlation coefficient closest to -0.3?

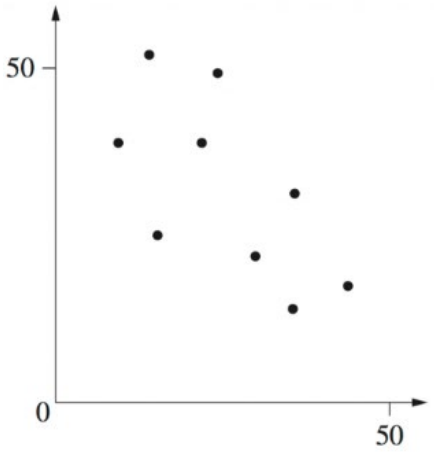
A.



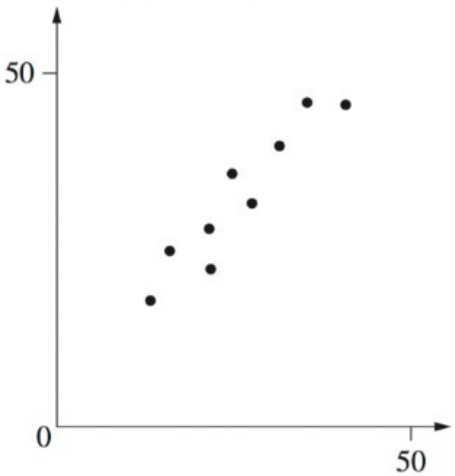
B.



C.



D.



8 A probability density function $f(x)$ is given by

$$f(x) = \begin{cases} \frac{1}{12}(8x - x^3), & 0 \leq x \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

The median, m, of this function satisfies the equation:

A. $-m^4 + 16m^2 - 6 = 0$

B. $m^4 - 16m^2 - 6 = 0.5$

C. $m^4 - 16m^2 + 24 = 0.5$

D. $m^4 - 16m^2 + 24 = 0$

9 How many solutions does the equation $|\sin 2x| = 1$ have for $0 \leq x \leq 2\pi$.

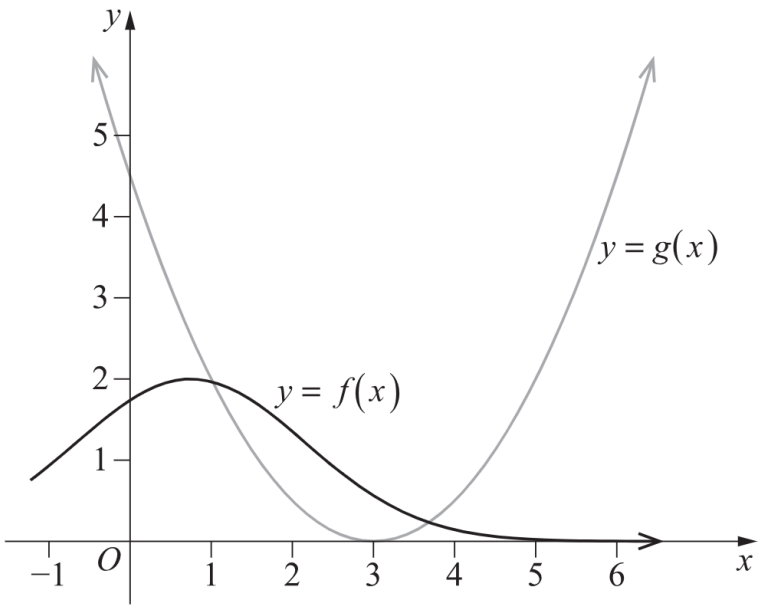
A. 1

B. 3

C. 4

D. 5

10 The graph shows two functions, $y = f(x)$ and $y = g(x)$



Define $h(x) = g\big(f(x)\big)$.
How many stationary points does $y = h(x)$ have for $0 \leq x \leq 5$.

A. 1

B. 2

C. 3

D. 4

End of Section I

Section II

90 marks
Attempt Questions 11–31
Allow about 2 hours and 45 minutes for this section

Write each response in the spaces provided. Extra writing space is provided at the back of this paper.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (3 marks)

Calculate the sum of the arithmetic series $10 + 17 + 24 + \cdots + 1074$. **3**

Question 12 (2 marks)

Evaluate $\int_0^{\frac{\pi}{2}} \cos(x)\, dx$. **2**

Question 13 (6 marks)

Steven picks two notes at random from five $5, three $10 and two $20 notes.

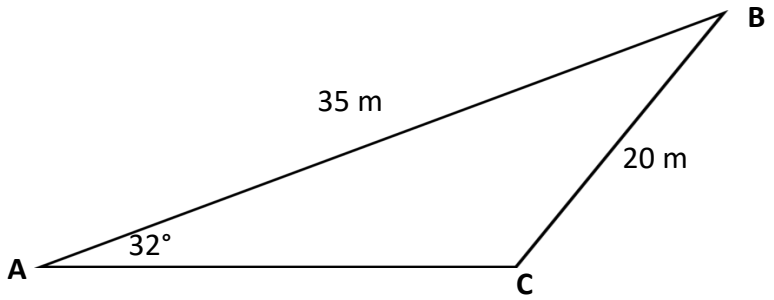
(a) What is the probability that Steven has picked a total of $20 or more? **2**

(b) Given that at least one of the notes is a $10 note, what is the probability that Steven has picked a total of $20 or more? **1**

(c) By determining the modal value, qualitatively compare the modal value of this experiment with the expected value. **3**
Give a reason to justify your comparison. (You do NOT have to calculate the expected value)

Question 14 (5 marks)

The diagram shows a triangle ABC where $AB = 35\text{m}$, $BC = 20\text{m}$ and angle $BAC = 32°$.



(a) Find the size of the obtuse angle ACB correct to the nearest degree. **3**

(b) Hence, or otherwise, find the bearing of C from B **2**

Question 15 (3 marks)

Find $\int \frac{x^2}{5 - x^3}\, dx$. **3**

Question 16 (3 marks)

(a) Differentiate $y = xe^{5x}$. **1**

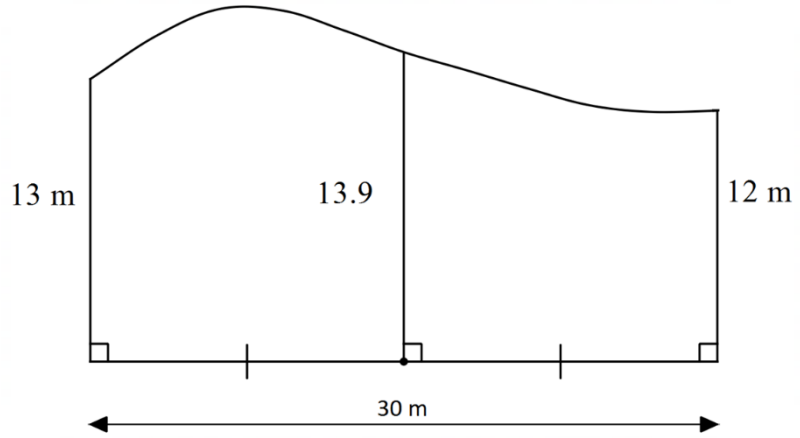
(b) Hence, find the exact value of $\int_0^3 e^{5x}(20x + 4)dx$. **2**

Question 17 (2 marks)

Prove that $(secx + tanx)(secx - tanx) = 1$ **2**

Question 18 (4 marks)

A park, which runs along a creek, has an irregular boundary line.



a. Use two applications of the Trapezoidal Rule to estimate the area of the park, to the nearest square metre. **2**

.....

.....

.....

.....

.....

.....

b. Identify one similarity and one difference between the process of using the trapezoidal rule and the process of integration. **2**

.....

.....

.....

.....

.....

Question 19 (6 marks)

A new species of bird is introduced to an ecosystem. The population of the species can be modelled by $P = 30 - 25(1.8)^{-0.3t}$, where P is the population of that species, measured in thousands, t years after it is introduced.

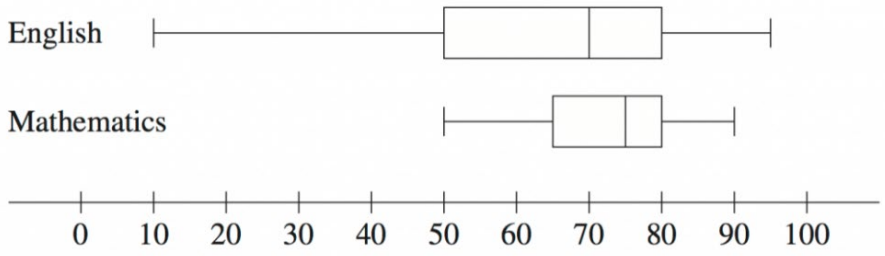
a. What is the population of this species 5 years after it is introduced? **1**

b. At what rate is the population increasing 5 years after it is introduced? **2**

c. How long after the species is introduced will the population reach 27 000? (Answer to the nearest year) **3**

Question 20 (7 marks)

The test results in English and Mathematics for a class were recorded and displayed in the box-and-whisker plots.



a. What was the median mark in the English test? **1**

.....

b. What was the interquartile range for the English test? **1**

.....

.....

c. Is the lowest score in the English test an outlier? Justify your answer with appropriate mathematical working. **2**

.....

.....

.....

.....

d. Compare and contrast the two data sets to determine which test was more difficult. In your answer refer to measures of centre, spread and shape. **3**

.....

.....

.....

.....

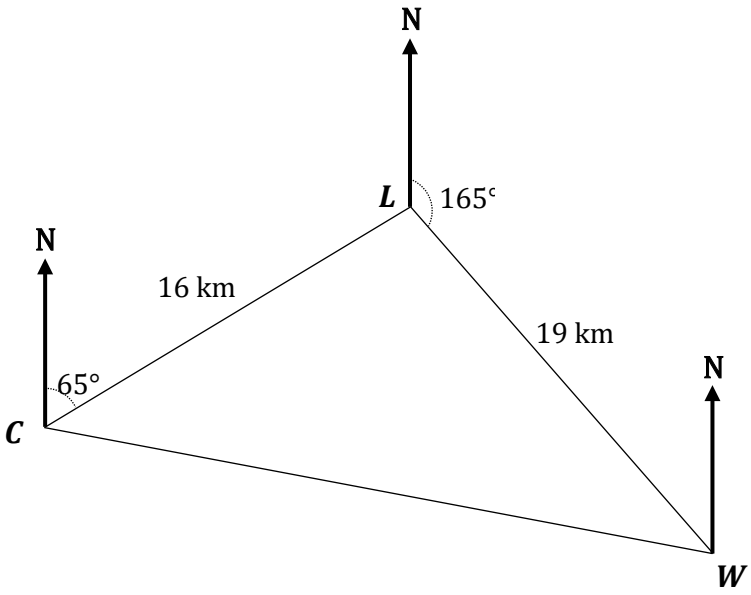
.....

.....

.....

Question 21 (3 marks)

A hiker decides to leave a campsite (C) to visit some natural landmarks in the area. The hiker walks 16 km on a bearing of 65° to see a lake (L). The hiker then turns and continues walking 19 km on a bearing of 165° to reach a waterfall (W).



a. Show that the size of $\angle CLW$ is 80°. **1**

.....

.....

.....

b. Calculate the distance between the waterfall and the campsite, correct to the nearest kilometre. **2**

.....

.....

.....

.....

Question 22 (3 marks)

A rational function $f(x)$ has the following properties: **3**

- The horizontal asymptote of its graph is $y = 0$
- The vertical asymptotes of its graph are $x = -2$ and $x = 2$

The table below shows the first and second derivates within certain regions and at $x = 0$.

	$x < -2$	$-2 < x < 0$	$x = 0$	$0 < x < 2$	$x > 2$
$f(x)$			1		
$f'(x)$	< 0	< 0	0	> 0	> 0
$f''(x)$	< 0	> 0	> 0	> 0	< 0

Sketch $y = f(x)$, using the information provided.

Question 23 (4 marks)

A probability density function is defined as:

$$f(x) = \begin{cases} k\sin(\pi x), & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

a. Show that $k = \frac{\pi}{2}$. **2**

b. Hence, find $P(X \leq 0.5)$. **2**

Question 24 (2 marks)

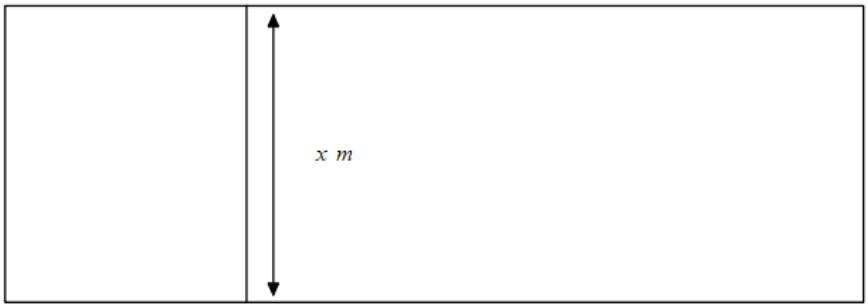
For the series $S(x) = \log_{10} x^2 + \log_{10} x^4 + \log_{10} x^6 + \cdots + \log_{10} x^{2n}$:

a. Show that $S_n = n(n+1)\log_{10} x$ **1**

b. Hence, or otherwise, write $\log_{10} 4 + \log_{10} 16 + \log_{10} 64 + \cdots + \log_{10} 4096$ in the form $a\log_{10} b$ where a and b are integers. **1**

Question 25 (4 marks)

A farmer has a large patch of land, on which he needs to build two fenced-in, rectangular areas as shown in the diagram below.



a. Given that the farmer has 312m of fencing, show that the total area of the two rectangles is given by $A = 156x - \frac{3x^2}{2}$. **1**

b. Hence, or otherwise, find the maximum area of the paddock, showing that it is the maximum area **3**

Question 26 (4 marks)

Consider the series $1 - \tan^2\theta + \tan^4\theta - \cdots$

a. Given that θ is $\frac{2\pi}{3}$, find the sum of the first four terms of the series. **2**
Give your answer as an exact value.

b. Find what values of θ in the interval $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ does the limiting sum of the series exist? **2**

Question 27 (3 marks)

Over a 24 hour period, the depth of water in a river is modelled by the function

$$h(t) = 10 + 6\sin\left(\frac{\pi t}{12}\right), \qquad 0 \leq t \leq 24$$

Where h is the depth of the water in metres, and t is the time, in hours, after 6am.

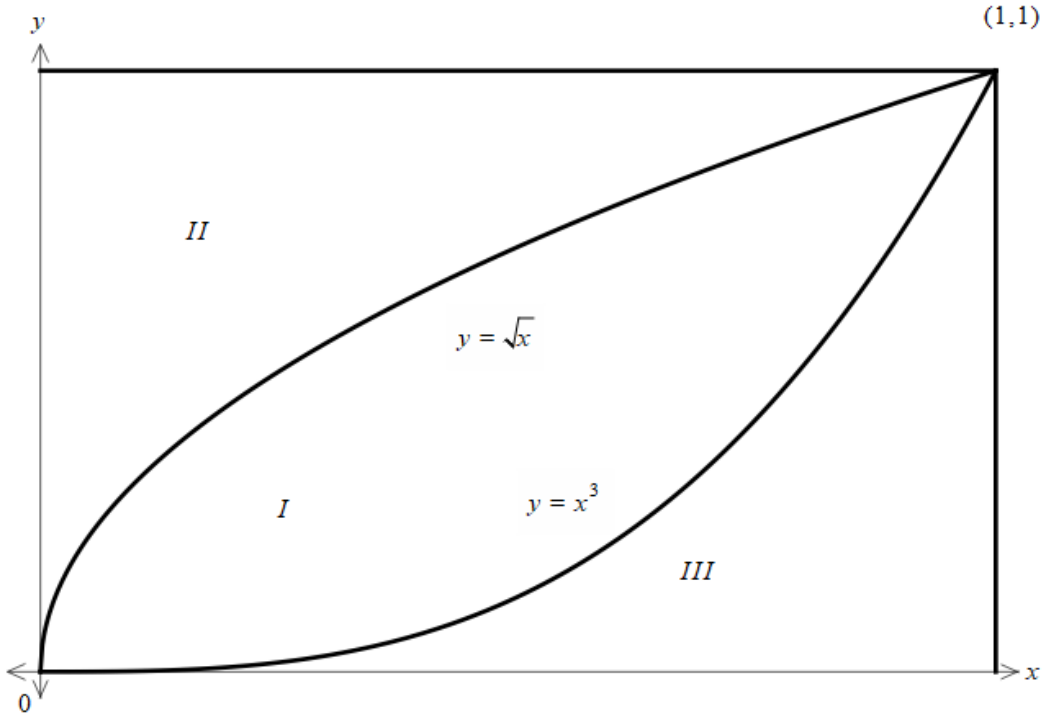
a. Find the maximum depth of the water in the river. **1**

b. At what times during the day is the water at a depth of 7m? **2**

Question 28 (7 marks)

The diagram below represents a target used for shooting. The entire target is within a unit square on the Cartesian Plane.

The target is divided into three non-overlapping regions, I, II and III by the curves $y = \sqrt{x}$ and $y = x^3$.



a. Find the areas of each of the three regions **3**

b. The scores for hitting regions I, II and III are 5, 10 and 15 points respectively. **2**

Calculate the expected value for a player who shoots once randomly at the target (assume they cannot miss the target entirely).

Give your answer correct to two decimal places.

.....

.....

.....

.....

.....

c. Barry shoots twice at the target and is told he scored 20 points. What is the probability that both of Barry's shots hit region II? **2**

.....

.....

.....

.....

.....

.....

.....

Question 29 (6 marks)

Consider the function $f(x) = \dfrac{\ln(x)}{x}$, for $x > 0$.

a. Show that the graph of $y = f(x)$ has a stationary point at $x = e$. **2**

b. Show that the stationary point at $x = e$ is a maximum. **2**

c. Hence, show that $e^x \geq x^e$ for all $x > 0$. **2**

Question 30 (5 marks)

A potato chip company uses a machine to fill bags of chips. When the machine is operating correctly, the weight of the bags are normally distributed with a mean of 90.0g and a standard deviation of 3.5g.

a. Assuming that the machine is working correctly, what is the probability that a random packet of chips will weigh more than 97g? **1**

.....

.....

b. To confirm that the machine is operating correctly, three chip packets are selected at random. The weights are: 91.2g, 94.6g and 86.9g. **2**

The machine needs to be checked when two or more weights lie more than 1 standard deviation from the mean.

Does the machine need to be checked? Justify your answer with calculations.

.....

.....

.....

.....

.....

c. Probabilistic tests can give false positives. **2**
A false positive is when a test indicates that something is wrong when it is, in fact, functioning normally.
What are the chances that the method described in part b. is used on a correctly operating machine but still produces a false positive? Give your answer to four decimal places.

.....

.....

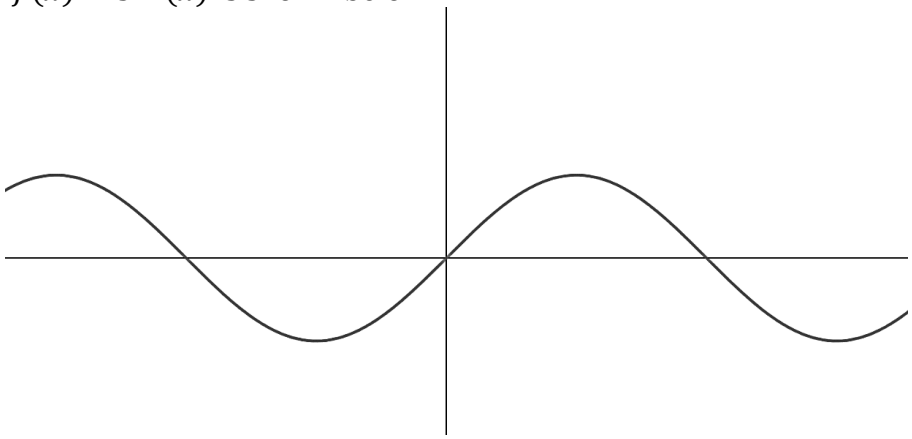
.....

.....

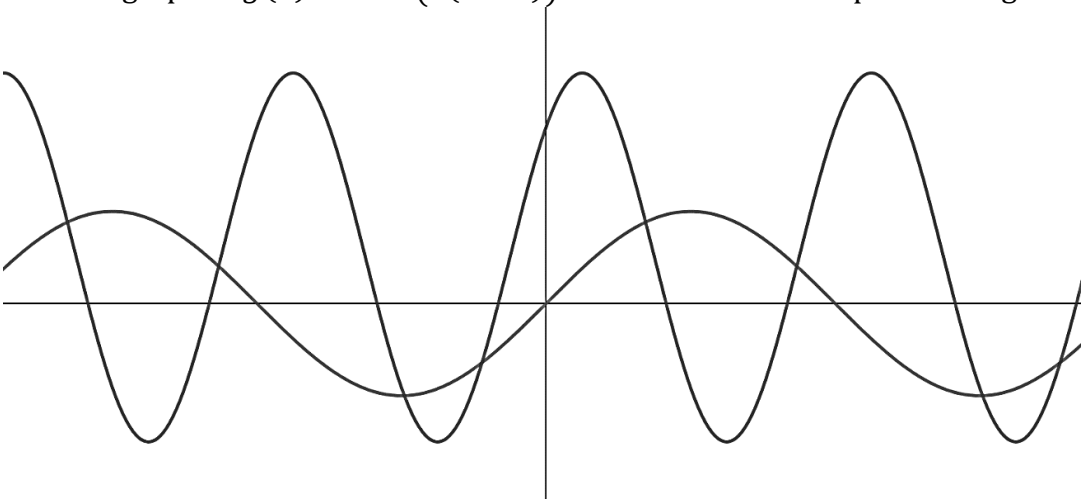
.....

Question 31 (8 marks)

The graph of $f(x) = \sin(x)$ is shown below:

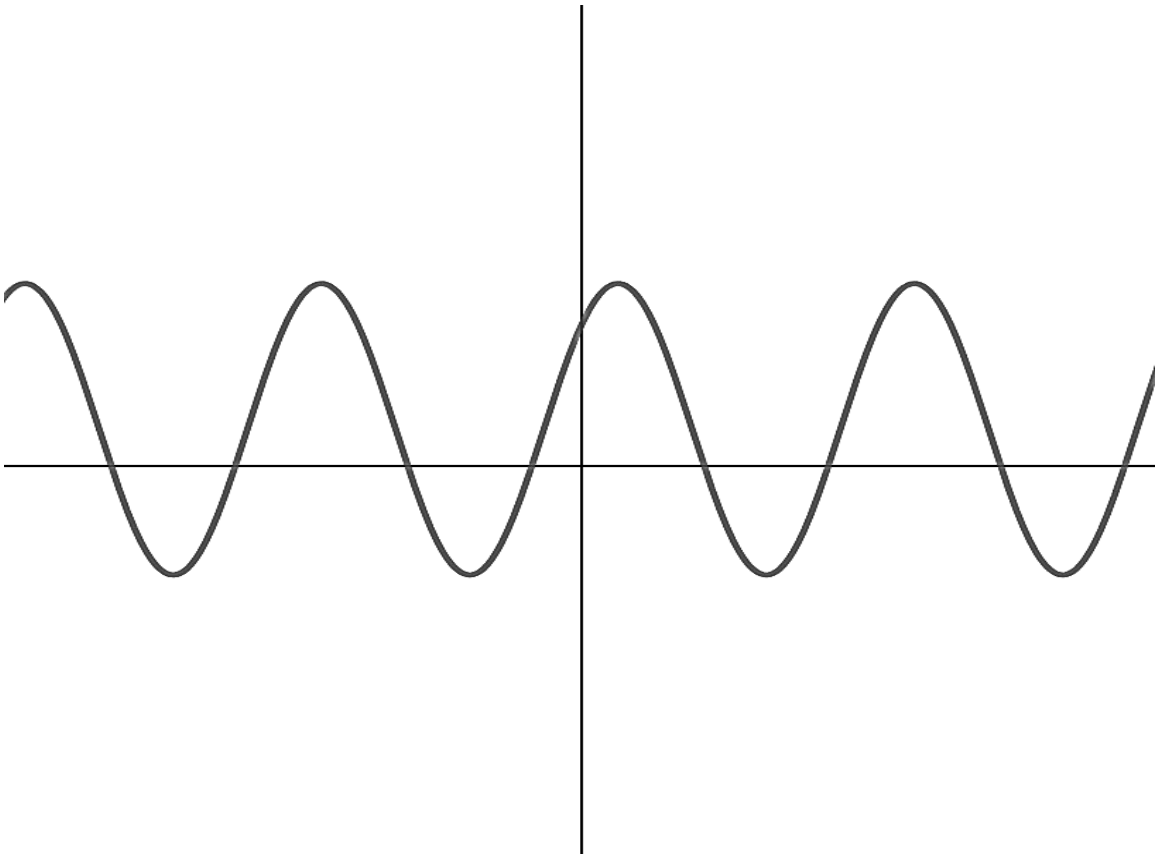


A transformed graph of $g(x) = a\sin\big(b(x+c)\big) + d$ is sketched on top of the original:



a. Given that a, b, c and d are all real numbers, suggest appropriate values for a, b, c and d with reasons given for each. You may refer to labelled annotations made on the diagram above. **4**

b. The same graph of $g(x) = a\sin\big(b(x+c)\big) + d$ is shown below. **4**
By first determining $g'(x)$ in terms of a, b and c, sketch the graph of $y = g'(x)$ on the same set of axes. You are not required to label your diagram.



.....

.....

.....

.....

.....

End of Examination

12MAA – Trial HSC Examination, 2023

Solutions and Marking Guidelines

Section I

1.	C
2.	A
3.	C
4.	B
5.	C
6.	B
7.	B
8.	D
9.	C
10.	A

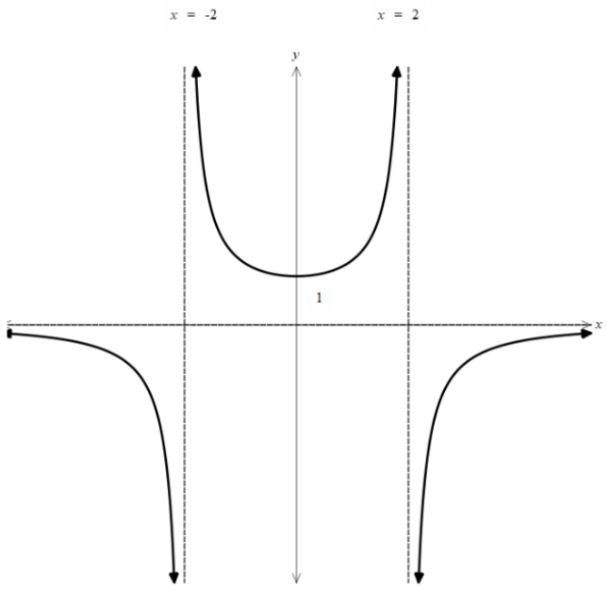
Section II

Question	Part	Sample Answer	Marking Guidelines
11		$d = 7\ a = 10$ $1074 = 10 + (n-1) \times 7$ $\frac{1064}{7} = n - 1$ $n = 153$ $S_n = \frac{n}{2}(a + l)$ $= \frac{153}{2}(10 + 1074)$ $= 82926$	3 – Correct Solution 2- Attempts to use the sum of an arithmetic series or equivalent OR Correctly finds n 1- Finds the values of a and d, or equivalent
12		$\int_0^{\frac{\pi}{2}} \cos x\, dx$ $= [\sin x]_0^{\frac{\pi}{2}}$ $= \sin\frac{\pi}{2} - \sin 0$ $= 1 - 0$ $= 1$	2- Correct solution 1- Finds the anti-derivative of $\cos x$ or equivalent
13	a)	$P(\geq 20) = 1 - \{P(two\ 5s) + P(5 + 10)\}$ $= 1 - \left(\frac{5}{10} \times \frac{4}{9} + \frac{5}{10} \times \frac{3}{9} + \frac{3}{10} \times \frac{5}{9}\right)$ $= 1 - \frac{5}{9}$ $= \frac{4}{9}$	2- Correct answer 1- Finds a relevant compound probability correctly **(recognising dependent events)** OR Finds 45 total outcomes
	b)	$P(10\ or\ 20) = \frac{4}{9}$	1- Correct answer

	c)	The modal (most likely) value is two \$5 notes = \$10. This is also the lowest possible value. As higher values are possible and have non-zero probabilities, the expected value must be greater than \$10.	3- Finds modal value and correctly justifies expected value as higher. 2- Finds modal value and correctly states expected value as higher. OR Correctly justifies expected value as higher than modal value 1- Finds modal value OR States expected value as higher than modal value
14	a)	For an acute reference angle D; $\frac{\sin D}{35} = \frac{\sin 32}{20}$ $D = \sin^{-1}\frac{35 \times \sin 32}{20}$ $\approx 68°$ $C = 180 - 68$ $= 112°$ $(nearest\ degree)$	3- Correct solution 2- Calculates acute angle or equivalent merit 1- Substitutes into sine rule correctly or equivalent
	b)	Bearing B from A =58° $B = 180 - 32 - 112$ (angle sum) $= 36°$ Bearing of C from B $= 360 - 122 - 36$ $= 202°$	2- Correct solutions 1- Finds a relevant bearing or equivalent merit
15		$\int \frac{x^2}{5 - x^3} dx$ $= -\frac{1}{3}\int \frac{-3x^2}{5 - x^3} dx$ $= -\frac{1}{3}\ln\lvert 5 - x^3\rvert + C$	3- Correct solutions 2- Applies appropriate formula, without absolute values and/or constant of integration 1- Adjusts expression appropriately or equivalent
16	a)	$y = xe^{5x}$ $\frac{dy}{dx} = 5xe^{5x} + e^{5x}$ $= e^{5x}(1 + 5x)$	1- Correct derivative

	b)	$\int_0^3 e^{5x}(20x+4)dx$ $= \int_0^3 4e^{5x}(5x+1)dx$ $= 4\int_0^3 e^{5x}(5x+1)dx$ $= 4[xe^{5x}]_0^3$ $= 4\left(3e^{15} - (0e^0)\right)$ $= 12e^{15}$	2- Correct solution 1- Attempts to use part a) or equivalent
17		$LHS = (\sec x + \tan x)(\sec x - \tan x)$ $= \sec^2 x - \tan^2 x$ $= \frac{1}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}$ $= \frac{1-\sin^2 x}{\cos^2 x}$ $= \frac{\cos^2 x}{\cos^2 x}$ $= 1, As\ required$	2- Correct proof 1- Uses identities for $\sec x$ and $\tan x$ or equivalent.
18	a)	$h = 15\ m$ $A_1 = \frac{15}{2}(13 + 13.9)$ $= \frac{807}{4} m^2$ $A_2 = \frac{15}{2}(13.9 + 12)$ $= \frac{777}{4} m^2$ $A = \frac{807}{4} + \frac{777}{4}$ $= 396m$	2- Correct answer 1- Correctly finds area of one trapezium
	b)	Similarities may include: • Both can be used to find an area • Both split a large area into smaller pieces • Both become more accurate as the pieces get smaller Differences may include: • Integration is derived from rectangles as opposed to trapeziums • Integration takes the limit as width goes to zero whilst trapezoidal rule have real widths • You need a defined function in order to integrate	2- Both a similarity and difference identified 1- Similarity or difference identified
19	a)	$P = 30 - 25(1.8)^{-0.3\times 5}$	1- Correct answer

		$= 19.647833 \ldots \ thousand$ $= 19648$	
	b)	$\frac{dP}{dt} = -25(1.8)^{-0.3t}(-0.3)(\ln 1.8)$ When $t = 5$ $\frac{dP}{dt} = -25(1.8)^{-1.5}(-0.3)(\ln 1.8)$ $=1.825\ldots$ Therefore at $t = 5$ the population is increasing by 1825 per year	2- Correct solution 1- Finds derivative of P correctly OR Substitutes into an incorrect derivative of P
	c)	$27 = 30 - 25(1.8)^{-0.3t}$ $\frac{-3}{-25} = 1.8^{-0.3t}$ $\ln\left(\frac{3}{25}\right) = -0.3t \times \ln 1.8$ $t = \frac{\ln\left(\frac{3}{25}\right)}{-0.3 \ln 1.8}$ $= 12.02$ $\approx 12 \ years$	3- Correct solution 2- Use the log function to solve the equation 1- Makes $1.8^{-0.3t}$ the subject of the equation or equivalent
20	a)	70	1- Correct answer
	b)	$\text{IQR} = 80 - 50 = 30$	1- Correct answer
	c)	Lower Limit$=Ql - 1.5 \times IQR$ $= 50 - 1.5 \times 30$ $= 5$ $10 > 5$ $\therefore \ The \ lowest \ score \ 10 \ is \ not \ an \ outlier \ since \ it \ is \ greater \ than \ the \ lower \ limit \ of \ 5.$	2- Correct answer with working 1- Correctly calculates lower limit for English
	d)	Mathematics has a higher median than English, indicating that the middle student performed better in Mathematics. The Mathematics results were also much more tightly grouped with an IQR of 15 compared to 30 for English. English is also negatively skewed with a lot of students scoring poorly compared to the median. All of these factors indicate the English exam was more difficult, even though the maximum English mark was higher than the maximum Mathematics mark	3- Correct conclusion with reference to centre, spread and skewness. 2- Correct conclusion with reference to two of centre, spread and skewness 1- Some relevant information
21	a)	$\angle CLW = 360 - (180 - 65) - 165$ $= 80°$	1- Correct answer
	b)	$CW^2 = 16^2 + 19^2 - 2 \times 16 \times 19\cos 80°$ $= 22.6146 \ldots$ $CW \approx 23 \text{ km}$	2- Correct answer 1- Uses cosine rule with at least one correct value

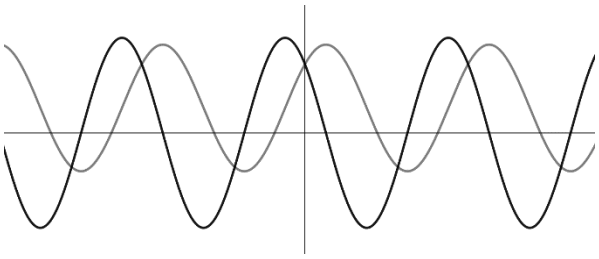
22			3- Provides correct solution, showing all essential features 2- Shows asymptotes on graph and 3 correct points from table 1- Shows asymptotes on graph or 1 correct point from table
23	a)	$\int_0^1 ksin\,(\pi x)$ $= \frac{k}{\pi}\int_0^1 \pi sin\,(\pi x)$ $= -\frac{k}{\pi}[\cos(\pi x)]_0^1$ $= -\frac{k}{\pi}(\cos\pi - \cos 0)$ $= \frac{2k}{\pi}$ As f(x) is a probability density function: $\int_0^1 ksin\,(\pi x) = 1$ $\frac{2k}{\pi} = 1$ $k = \frac{\pi}{2}\ as\ required.$	2- Correct solution 1- Finds anti-derivative of $ksin(\pi x)$ OR Indicates that the integral over the domain must be equal to 1

	b)	$P(X \leq 0.5) = \int_0^{0.5} \frac{\pi}{2} sin\,(\pi x)$ $= \frac{1}{2}[-\cos \pi x]_0^{0.5}$ $= \frac{1}{2}(-\cos\frac{\pi}{2} + \cos 0)$ $= \frac{1}{2}(-0 + 1)$ $= \frac{1}{2}$	2- Correct solution 1- Correctly establishes $P(X \leq 0.5) = \int_0^{0.5} \frac{\pi}{2} sin\,(\pi x)$
24	a)	$S_n = \frac{n}{2}(a + l)$ $= \frac{n}{2}(logx^2 + logx^{2n})$ $= \frac{n}{2}(2\log x + 2n\log x)$ $= n(\log x + n\log x)$ $= n(n+1)\log x$	1- Correct answer
	b)	$4096 = 2^{12} = 2^{2\times 6}$ Therefore $n = 6, x = 2$ $S_n = n(n+1)\log x$ $= 6(6+1)\log 2$ $= 42\log 2$	1- Correct answer
25	a)	$A = x \times \left(\frac{312 - 3x}{2}\right)$ $= 156x - \frac{3x^2}{2}$	1- Correct answer
	b)	$\frac{dA}{dx} = 156 - 3x$ Stationary point when $\frac{dA}{dx} = 0$ $156 - 3x = 0$ $x = \frac{156}{3}$ $= 52$ $\frac{d^2A}{dx^2} = -3$ Maximum when second derivative <0, which is always true. At $x = 52$ $A = 156 \times 52 - \frac{3(52)^2}{2}$ $= 4056m^2$	3- Provides correct solution 2- Finds correct stationary value and justifies it is a maximum 1- Finds correct stationary value and sets $\frac{dA}{dx} = 0$

26	a)	$S_n = \frac{a(1-r^n)}{1-r}$ $r = -\tan^2\frac{2\pi}{3}$ $= -\left(\sqrt{3}\right)^2$ $= -3$ $S_n = \frac{1(1-(-3)^4)}{1--3}$ $= -20$	2- Correct solution 1- Finds r or equivalent merit
	b)	$\|r\| < 1$ $\|\tan^2\theta\| < 1$ $\tan^2\theta < 1$ $tan\theta < 1 \; or \; \tan\theta > -1$ $\theta < \frac{\pi}{4} \; or \; \theta > -\frac{\pi}{4}$ $\therefore -\frac{\pi}{4} < \theta < \frac{\pi}{4}$	2- Correct solution 1- Recognition that the common ratio must have absolute value < 1
27	a)	$10 + 6 = 16m$	1- Correct Solution
	b)	$7 = 10 + 6\sin\frac{\pi t}{12}$ $\sin\frac{\pi t}{12} = -\frac{1}{2}$ $\frac{\pi t}{12} = \frac{7\pi}{6}, \frac{11\pi}{6}$ $t = 14, 22$ Therefore, the water is at a depth of 7m at 8pm and 4am	2- Finds both t values correctly 1- Solves to $\sin\theta = -\frac{1}{2}$ or equivalent merit

28	a)	$Area(III) = \int_0^1 x^3\ dx$ $= \frac{x^4}{4}\Big\vert_0^1$ $= \frac{1}{4}$ $Area(I) = \int_0^1 \sqrt{x} - x^3\ dx$ $= \frac{2}{3}\sqrt{x^3} - \frac{x^4}{4}\Big\vert_0^1$ $= \frac{5}{12}$ $Area(II) = 1 - \frac{1}{4} - \frac{5}{12}$ $= \frac{1}{3}$	3- Correct solution 2- Two correct areas 1- One correct area
	b)	$E(X) = \sum xp(x)$ $= 5 \times \frac{5}{12} + 10 \times \frac{1}{3} + 15 \times \frac{1}{4}$ $= 9.16666\ldots$ ≈ 9.17	2- Correct solution with rounding 1- States formula for expected value
	c)	$P(20) = P(10,10) + P(5,15)$ $= \frac{1}{3} \times \frac{1}{3} + 2 \times \frac{5}{12} \times \frac{1}{4}$ $= \frac{23}{72}$ $P(10,10\vert 20) = \frac{1}{9} \div \frac{23}{72}$ $= \frac{8}{23}$	2- Correct answer 1- Correctly finds probability of scoring 20 points OR Finds probability of both shots hitting region II
29	a)	$f(x) = \frac{\ln(x)}{x}$ $f'(x) = \left(\frac{1 - \ln x}{x^2}\right)$ Stationary points when $f'(x) = 0$ $1 - \ln x = 0$ $\ln x = 1$ $x = e$	2- Correct solution 1- Computes $f'(x)$ correctly or equivalent progress
	b)	$f''(x) = \frac{-3x + 2x\ln x}{x^4}$ $f''(e) = \frac{-3e + 2e}{e^4} < 0$ Therefore the curve is concave down at $x = e$. Therefore this is a maximum	2- Correct solution 1- Finds the second derivative correctly OR

			Attempts to sketch curves of $y = \ln x$ in relation to $y = x$
	c)	$f(x)$ has a maximum at $\left(e, \frac{1}{e}\right)$ $\therefore \frac{\ln x}{x} \leq \frac{1}{e}$ $e \ln x \leq x$ $\ln x^e \leq x$ $x^e \leq e^x$	2- Correct solution 1- Observes that $\frac{\ln x}{x} \leq \frac{1}{e}$
30	a)	97 has a z-score of $+2$. Therefore, chance if <97 is $5\% \div 2 = 2.5\%$	1- Correct solution
	b)	Limits are: $< 90 - \sigma \; OR > 90 + \sigma$ $< 86.5g \qquad > 93.5g$ Only the 94.6g sample falls out of this range, so the machine does not need to be checked.	2- Correct solution with working for upper and lower bounds 1- Correct conclusion without working or vice versa
	c)	$P(at\ least\ 2\ with > \lvert\sigma\rvert\ variation)$ $= 1 - P(0\ or\ 1\ with > \lvert\sigma\rvert variation)$ As 68% of scores fall withing 1 std. dev: $= 1 - (0.68^3 + 3 \times 0.68^2 \times 0.32)$ $= 0.2417$	2- Correct solution 1- Makes a relevant calculation with empirical values
31	a)	b dilates horizontally. As $g(x)$ has a smaller period, by around 1.5, $b \approx 1.5$ a dilates vertically. As $g(x)$ has a larger amplitude by a factor of 2, thus $a \approx 2$ c and d translate horizontally and vertically respectively. The curve has been moved up and to the left. $c \approx 0.5, d \approx \frac{\pi}{4}$	4- Reasonable values for all pronumerals, with links to how each transforms the function 3- three values correct with links to transformations 2- All reasonable values, with some links OR Two correct values with links to transformations 1- One correct value and link to transformations OR Multiple reasonable values Note: Multiple possible correct values for the translations

	b)	$g(x) = a\sin\big(b(x+c)\big) + d$ $g'(x) = ab\cos(b(x+c))$ Key features: Amplitude is larger as a and b>1 No constant, so now equal amplitude above and below the x-axis Going from sin>cos shifts the graph by $\frac{\pi}{2}$ 	4- Correct differentiation and graph 3- Correct differentiation and inclusion of two key features 2- Correct differentiation and inclusion of one key feature OR Differentiate with one error, and multiple key features accurately shown 1- Correct differentiation OR Recognition that the derivative will be a cos curve